PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES
Volume 5 JUNE 15, 1919 Number 6

THE THERMO-ELECTRIC EQUATION $P = TdV/dT$ ONCE MORE

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Communicated, March 27, 1919

At the Philadelphia meeting of the Academy in November, 1917, I questioned the validity of this equation as commonly understood, P being taken as the ordinary Peltier effect and V the Volta effect between any two metals. But I did not make my point of objection entirely clear and wish now to try again; for the matter is important.

It appears that Kelvin, who derived this equation from a course of theoretical reasoning, did not regard P as necessarily the Peltier effect only, believing that it might include some other, hitherto unknown, reversible heat effect accompanying movement of electric charge from one metal to another. O. W. Richardson, arriving at the same equation by a different course of reasoning, held P to be simply the Peltier effect. My colleague Professor Bridgman, repeating with some modifications of his own the argument of Kelvin and also that of Richardson, came at first to a conclusion sustaining that of Richardson, and submitted his unpublished paper to me for criticism.

I maintained that Kelvin's broader interpretation of the P was probably the correct one and ventured the suggestion that ionization and re-association within the metals according to the mass-law of equilibrium between free electrons and metal ions, furnished the reversible heat effect, additional to the ordinary Peltier effect, that Kelvin saw the need of. Bridgman felt, however, that I had not disposed of Richardson's argument, and accordingly I wish now to revise, but not to withdraw, my criticism.

The passage which I quoted from p. 28 of his *Emission of Electricity from Hot Bodies* ended with the equation

$$dS = \frac{1}{T}[d(nv\phi) + pdv],$$

in which dS is change of entropy of the system and "φ is the change in the energy of the system which accompanies the transference of each electron

from the hot body to the surrounding enclosure." Now I was wrong in objecting to this particular passage, for just here Richardson is speaking of an insulated piece of metal which is imagined to give off electrons that do work as a gas on a moving piston; and for such a 'virtual' operation his statement is doubtless true. But in his *experiments* he is really taking off a stream of electrons in the gas state from the metal, and an equal stream is constantly entering the metal by conduction, yet he assumes that the same Φ which occurs in the equation above quoted, for his case of virtual emission from an insulated body, holds as the heat of emission for his actual case.

It was this actual case, of electric flow, that I had in mind when I offered my criticism that his equation for dS fails when not only heat but substance also, that of the incoming electrons, is added to the system during the operation under consideration. I believe that this objection holds.

Bridgman, missing my point through my lack of precision in stating it, studied Richardson's argument again and presently made for himself the same discovery that I had made, so that he and I are now in agreement regarding the inaccuracy of Richardson's reasoning. He finds, moreover, that, when this inaccuracy is corrected, Richardson's line of argument leads to precisely the same result as Kelvin's.

Accordingly Bridgman now writes P' instead of P in the equation in question, meaning by P' the total reversible heat effect that accompanies a virtual movement of charge from one plate to another of a condenser made of different metals, though he may not subscribe entirely to my theory as to the action of ionization and re-association within the metals.

I hope that Professor Bridgman's paper, dealing with thermo-electricity in a broad way, will be published before long.

ON THE X-RAY ABSORPTION WAVE-LENGTHS OF LEAD ISOTOPES

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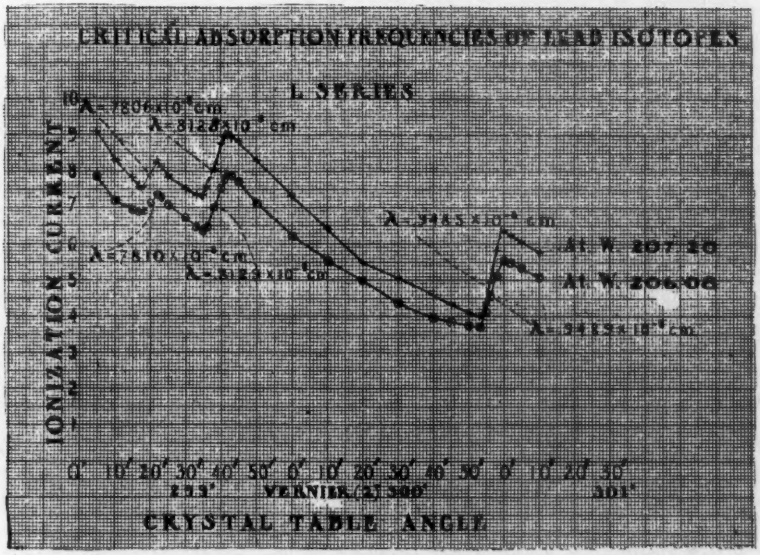
Communicated by T. Lyman, April 5, 1919

Researches on the transformations of radioactive substances have lead to the conclusion that chemical elements exist which have the same atomic number but different atomic weights. Such chemical elements have been called isotopes. In the reactions of ordinary chemical analysis isotopes behave in identically the same manner. It has not been found possible to separate isotopes from each other by means of purely chemical processes; although it seems probable that, since the atomic weights of isotopes differ from each other, they will act somewhat differently in those phenomena in which the mass of the atom enters as a factor. The ordinary line spectra of

chemical isotopes, also appear to be identical to a very high degree of precision, and recently Siegbahn and Stenström found no difference between their emission spectra in what is called the L series of X-rays.

The object of the research reported in this note has been to investigate the X-ray absorption spectra of chemical isotopes. In general each chemical element, except perhaps those of low atomic numbers, has several critical absorption frequencies, one connected with its K series and three connected with its L series of X-rays. These critical absorption frequencies mark points in the X-ray spectrum where sharp changes in the absorption of X-rays by the chemical element occur. The chemical element absorbs X-rays of higher frequency than the critical frequency to a much greater extent than it does X-rays of lower frequency.



In measuring the critical absorption frequencies of lead isotopes we have used the X-ray spectrometer described in the *Physical Review* for December 1917, page 624. A calcite crystal reflected the X-rays whose wave-length is given by the equation

$$\lambda = 2a \sin\theta = 6.056 \sin\theta \times 10^{-8} \text{ cm.}$$

(where θ is the grazing angle of incidence) into an ionization chamber. The ionization method of detecting the reflected beam of X-rays is far superior to the photographic method, both because, if properly used, it requires no correction for the penetration of the X-rays into the crystal, and also because it gives an estimate of the magnitude of the absorption.

The absorbing screen of lead was placed between the X-ray tube and the spectrometer. Professor Richards' laboratory kindly furnished us with the specimens of lead salts. Professor Richards has made accurate measurements of the atomic weights of lead isotopes, and the values he obtained for the two specimens we used were 207.20 for the ordinary lead and 206.08 for the radioactive lead. These differ from each other by more than $\frac{1}{2}\%$.

In our experiments the X-rays came from a molybdenum target tube of the Coolidge type, and a constant difference of potential amounting to about 36,000 volts drove a current of 2 milliamperes through it.

The curves in the figure represent the ionization currents as functions of the readings of one of the verniers attached to the crystal table.

The three sharp drops in each curve correspond to the three critical absorption wave-lengths belonging to the L series of X-rays of each specimen of lead respectively. To get the grazing angles of incidence to substitute in the above formula for the wave-length we measured from the centres of the drops to the zero, 291° 55′ 40″, the value of which has been corrected for eccentricity.

The values of the wave-lengths, etc., have been collected together in the following table.

ABSORBING SCREEN	ATOMIC WEIGHT	$\lambda \times 10^{8}$ CM.	$\lambda \times 10^{8}$ CM.	$\lambda \times 10^{8}$ CM.
Ordinary lead	207.20	0.9485	0.8128	07.806
Radioactive lead	206.08	0.9489	0.8129	0.7810

The grazing angles of incidence can be estimated to within about 30″ of arc, which means that the wave-lengths are correct to within about 0.1%. Corresponding wave-lengths for the two specimens in the above table do not differ from each other by as much as 0.1%, and, therefore, the critical absorption wave lengths of the isotopes of lead are identical to within the limits of error of the experiments.

The magnitude of the characteristic absorption can be estimated from the drops in the curve. These depend, of course, upon the thickness of the absorbing layer, and its measurement is very much less accurate than that of the wave-length. It appears, however, that the relative change of absorption at the three critical wave-lengths is about the same for each isotope of lead.

SOME ORIENTING EFFECTS OF MONOCHROMATIC LIGHTS OF EQUAL INTENSITIES ON FUCUS SPORES AND RHIZOIDS

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Communicated by J. M. Coulter, April 11, 1919

One of the most striking of the biological phenomena resulting from the action of light on organisms is the orientation of the first cleavage plane of germinating spores by unilateral illumination. Whenever such illumination is sufficiently intense the first cross wall forms perpendicular to the direction of the incident light. This phenomenon has been demonstrated in Equisetum, Puccinia, Fucus and some other algae[4,8,9,10,11] together with the fact that the cell on the shaded side of the spore becomes the rhizoidal cell irrespective of gravity. Thus in these and related forms the polarity of the plant is established by the direction of light stimuli.

The power of light waves to so orient the plant is, without doubt, the power to orient the spindle of the first dividing nucleus. The mechanics of such reactions may long remain unknown; but we have a suggestive and possibly the ultimate explanation in Child's[2] metabolic gradient theory. He has demonstrated in many marine plants and in some of the lower animals the existence of the so-called 'axial gradients.' By an axial gradient is meant the decreasing rate of metabolic processes from the apical to the basal end. We may suppose that such a gradient is produced within a germinating spore whenever there is sufficient difference in the amount of light energy received on two opposite sides to produce the requisite difference in the rate of the oxidation processes along the line of direction of illumination. If Child's supposition is correct, the cell on the shaded side of the spore becomes the rhizoidal cell by virtue of the fact that the least rapid rate of the oxidation reactions along the gradient determines the basal end, the most rapid the apical end, of the organism.

The purpose of the present investigation was to study the power of pure monochromatic lights to establish the polarity of the germinating spores of *Fucus inflatus*, and also to answer several questions concerning the negative phototropism of the young rhizoids; viz., the determination of the exact wave lengths responsible for the phenomenon; the relative importance of the quality and quantity factors in the illumination or the role of intensity of illumination apart from the kind of light; and whether all effective monochromatic lights produce the same result as white light.

To obtain the monochromatic light, seven Wratten filter screens were used, each transmitting a narrow range of wave lengths but altogether embracing the whole of the visible spectrum. The wave lengths to which each screen was transparent were determined by testing the light transmitted by each with

a direct-vision spectroscope with a wave length scale attached. These screens were then fitted as windows in the ends of boxes painted black on the inside. Culture dishes were made by cementing together microscope slides, so that the light entering the boxes through the screens fell on a flat side of the dish and thus entered the water normally with a minimum loss from reflection and refraction.

The electric arc was used as the source of light wherever possible because it gives all the desired wave lengths, with the result that the whole set of screens could be used in the same exposure, insuring for all the boxes identical conditions of temperature and duration of illumination.

The spectroscopic analysis of the light transmitted by the screens gave the exact wave lengths which would act on the cultures behind them. The next step was to devise a means of making the intensities of the lights acting in each box equal so as to eliminate that most important, and hitherto largely overlooked, factor in light reactions. This done, variations in results obtained behind different screens might safely be attributed to differences in the quality of the stimulus. There have been several methods devised by which the relative intensities of monochromatic lights can be measured and made equal.[1,3,5,6,7] But all of these involve special apparatus not available for use in this investigation; so a simpler method was devised whereby the relative intensities of the lights transmitted by the Wratten filter-screens were measured by means of a thermopile and galvanometer and made equal by varying the distances of the dark boxes from the electric arc such that at these distances the deflections of the galvanometer, when the thermopile was exposed in turn to the light behind each filter screen, were equal. This distance was also measured with a piece of white glass as a filter screen which represented the removal of the control culture from the arc. The instruments used in this energy calibration were a Hilger thermopile and a d'Arsonval galvanometer.

It seems necessary on account of the questions which have been raised during the course of this work, to state here that the thermopile is equally sensitive to the energy of the red and of the violet ends of the spectrum, and is, therefore, an accurate measure of the total amount of light acting behind each color screen. The difference between heat and light is only a matter of wave length. The thermopile measures light in terms of the electric current produced by the difference in temperature of the exposed and unexposed junctions; but it does so by virtue of the fact that the energy of whatever vibrations fall upon it, be they long and therefore heating in their physiological effect, or short and therefore perceived as light, is converted into heat energy upon being absorbed by the exposed junction of the thermopile. In other words, the light of the blue end of the spectrum produces an electromotive force much less than that of the infra-red but no less measurable.

Once these distances from the arc, at which the intensities of the light in each box are equal, are determined, the quantity of light energy can be varied

by multiplying or dividing all the distances by the same multiple and the intensities in all will still remain equal to each other. By means of a photometer the actual amount of light acting in each box can then be determined by measuring the intensity in candle meters behind the white glass control at the proper distance from the arc. Then from the law of inverse squares, viz., that the intensity of light per unit surface varies inversely as the square of the distance from the source, the absolute intensity at any distance from the arc can be computed. So a Sharpe-Millar photometer was used to measure the intensity of the naked arc at the distance of the white light control. But it was then necessary to correct the measurements so obtained for the absorption of light by the glass of the filters. This so-called absorption coefficient was obtained by measuring with a Lummer Brodhum photometer the intensity of a light both with and without a screen of clear glass equal in thickness to that of the filters. It was found that glass 1.5 mm. thick absorbed 12% of the light falling upon it. Therefore to obtain the intensity of the light actually entering each dark box, it was necessary to take 88% of the reading given by the photometric measurement of the unscreened arc at the previously determined distance of the white light control.

The following table lists the colored screens used with the wave lengths they transmitted and the distances from the arc at which they were placed to make the intensity of light behind each equal to 1800 meter candles. The lack of agreement between these values and the energy curve of the spectrum is due to the individual absorption of the filters and also to the fact that they do not all transmit the same number of wave lengths.

Table showing distances at which the intensities of light from an electric arc transmitted by Wratten light filters are equal

FILTER NUMBER	WAVE-LENGTHS IN ANGSTROMS	COLOR	DISTANCE FROM LIGHT	INTENSITY IN METER CANDLES
			cm.	
70	6600–7000	Red	320	1800
71	6200–6800	Red	275	1800
72	5900–6200	Orange	230	1800
73	5600–5900	Yellow	250	1800
74	5200–5600	Green	280	1800
75	4700–5200	Blue	250	1800
76	4000–4700	Violet	250	1800
Control	4000–7000	White	340	1800

To obtain the spores of *Fucus inflatus* for the experiments, the fruiting plants were collected at low tide, kept over night in damp newspapers, and the next morning were dried slightly by exposing them to the air for about half an hour. Then when the fruiting tips were submerged in sea water in the culture dishes, large numbers of eggs and sperms were extruded and settled to the bottom of the dish. After removing the piece of plant the culture dish was placed in one of the little racks made to fit in the dark boxes

behind the filter screens. The illumination of the cultures was continued six to eight hours, this time having been found more than enough to cause the first cleavage plane to be permanently oriented regardless of subsequent illumination.

In the experiments for which the naked arc was the source of light, the heating effect was so great that the spores were killed very quickly. The mercury vapor lamp was next used to obtain wave lengths of the blue end of the spectrum and a 1000 watt nitrogen filled Tungsten globe for the red. But as in the case of the electric arc, the Tungsten light killed the spores by the high temperature produced at the distances where it was necessary to place the cultures for a sufficiently intense illumination. With the mercury vapor lamp, however, positive results were secured. The wave lengths which were found to produce the orientation of the first cleavage plane such that all the first cross walls formed perpendicular to the direction of the incident rays, are those between 4000 and 5200 Ängstrom units. Behind the two other filter screens used with the mercury vapor lamp and transmitting wave lengths of 5200 to 5900 Ängstrom units, the spores germinated as if in darkness with the orientations of the first cleavage planes following no rule, and the rhizoids extending in all directions. However, the intensities of the lights behind these color screens were not equal when the mercury lamp was used because the shorter blue wave lengths predominated to so great a degree and hence produced greater intensities.

With regard to the phototropism of the young rhizoids, it was found that very weak white light, too weak to orient the cleavage planes, would cause the growing tips to turn sharply away from the source of light. With the intensity of illumination behind all the color screens equal to 1800 meter candles, only the blue and violet lights produced the phototropism. The other wave lengths at this intensity had no effect, the young rhizoids continuing in the direction in which they had started just as did those of the control in darkness. However when a more intense illumination was secured by placing the boxes in direct sunlight, the rhizoids behind the green filter, in addition to those behind the blue and violet ones, showed the same negative phototropism. This and subsequent experiments would lead us to believe that both quantity and quality, or intensity and wave length, are determining factors in the power of light stimuli to produce phototropisms.

In every culture of *Fucus inflatus* whether germinated in darkness or in strong unilateral light a most striking orientation of the first cross-wall with reference to adjacent spores appears. Wherever a group of spores are lying within about 0.2 mm. of each other, the first cleavage plane is perpendicular to the direction of the center of the group. The cell toward the interior invariably becomes the rhizoidal cell. This phenomenon was reported by Rosenvinge[9] in other species of *Fucus* and in *Ascophyllum*. For want of a better term I have called it *group orientation*. A study of the phenomenon was made to determine the strength of this stimulus, compared to that of

light, in its power to establish the orientation of the plant. It was at once very evident that for most spores the former stimulation is stronger when the spores are within a short distance of each other—0.2 mm. or often more—but beyond this distance, the chemical stimulus becomes too weak and only the light is able to determine the polarity of the plant. Only the comparatively isolated spores therefore show the orientation to light with the sources of illumination used here.

The phenomenon is very conspicuous in groups of 2, 3, or 4 eggs as well as in masses of 50 or 100. In these large groups it is made evident by the invariable rule that no rhizoid ever extends outward from a group. When two spores are within the distance through which the stimulus is effective, the first cleavage planes of the two are parallel and the rhizoids grow towards each other and often meet tip to tip. The groups of 5 or 6 often make symmetrical star-like designs when the rhizoids have grown and project beyond the group. The spores are more rarely affected in this way when the distance between them is over 0.3 mm. but the phenomenon is sometimes observed in spores as much as 0.5 mm. apart. Within a distance of 0.2 mm. there are practically no exceptions.

The relative sensitiveness of a spore towards light and towards this chemical (?) stimulus varies greatly for different spores. When cultures were placed in the window to get as strong a light stimulus as possible in order to determine at what distance from each other the eggs had to be not to show a greater sensitiveness towards the chemical stimulus than towards the light, it was found that this distance followed no rule, the spores showing the greatest individual differences. Of two spores lying within 0.3 mm. of each other one might be entirely oriented by the adjacent spore while the other, apparently like it, would show only the action of the light stimulus. In many cases two such spores would seem to show a resultant effect of the two stimuli so that both would be half turned towards each other with both rhizoidal cells showing a tendency to take a direction away from the light at the same resultant angle.

Rosenvinge ascribes this group orientation to a difference in the concentration of oxygen or of nutritive substances on the two sides of the spore. He thinks the rhizoid forms on the side toward the center of a group or towards another egg because the water on that side is less rich in the active substance than on the outer side as a result of their metabolism. Winkler[11] working with *Cystoseira barbata* found that a difference in oxygen concentration has no such effect. Apparently the phenomenon does not occur naturally in this species since his figure shows nothing but the effect of light. I have never seen a culture of *Fucus inflatus* with spores germinating so near each other, which showed only light orientation and not the group orientation. Almost invariably when the spores of this species germinate in such close proximity, light appears to have no power to establish the polarity of the plant.

The possibility that the group orientation is due to a polarity established by the position of the egg in the oogonium is suggested by finding many

groups of eight lying just as they escaped from the oogonial sac and conspicuously oriented with respect to each other. The fact that groups of ten or of two are as regularly oriented, would refute the suggestion; but in order to prove that the phenomenon is the result of a stimulus acting after the eggs leave the oogonium, a group of them were transferred to a watch crystal and mixed with the point of a needle until their relative positions were entirely changed. But when they germinated the characteristic orientation with respect to each other was found to be without an exception.

The only apparent explanation of the group orientation is that of a diffusion gradient of some substance emanating from a growing spore, or of some substance being used up by it. A continuation of the investigation of this problem will be an attempt to discover a substance which can so affect the dividing nucleus of the egg cell that its unequal distribution on the sides of the cell will orient the axis of the spindle. The effect of bubbling carbon dioxide and oxygen through cultures should be tried as being the most probable factors involved.

The substance or condition originating in the activity of adjacent spores which has so powerful an effect in orienting the first cleavage plane and in determining which cell shall become the rhizoidal cell has no power to cause any chemotropism of the rhizoids after they are started. No rhizoid has been found to have its direction modified by the presence of other spores adjacent to it. In the absence of any light stimulus the rhizoids continue in the direction that they take originally from the spore.

[1] Blaaw, A. H., *Rec. Travaux Bot. Néerlandais*, **5**, 1908, (209–372).
[2] Child, C. M., *Individuality of Organisms*, 1915.
[3] Day, E. C., *Bull. Mus. Comp. Zool.* **53**, 1911, (303–343).
[4] Farmer, J. B., and Williams, J. L., *Proc. Roy. Soc.*, **60**, 1896, (188–195).
[5] Kniep, G. and Minder, F., *Zs. Bot.*, **1**, 1909, (619).
[6] MacDougal, D. T. and Spoehr, H. A., *Science*, (N. S.), **45**, 1917, (616–618).
[7] Mast, S. O., *J. Comp. Neur. Psych.*, **17**, 1907, (99–179).
[8] Randolph, F. A., and Peirce, G. J., *Bot. Gaz.*, **40**, 1905, (321–350).
[9] Rosenvinge, M. L. K., *Revue Gen. Bot.*, **1**, 1889, (125–135).
[10] Stahl, E., *Ber. deuts. bot. Ges.*, **3**, 1885, (334–340).
[11] Winkler, H., *Ibid.*, **18**, 1900, (297–305).

ON THE MOST GENERAL CLASS L OF FRÉCHET IN WHICH THE HEINE-BOREL-LEBESGUE THEOREM HOLDS TRUE

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Communicated by E. H. Moore, April 15, 1919

§1 A class L of Fréchet[1] is a set of elements such that (1) if P is an element of L and $P_1, P_2, P_3 \ldots$ is a countable[2] sequence of elements belonging to L then the statement that P is the limit of the sequence $P_1, P_2, P_3, \ldots$

has a definite meaning and the question whether this statement is true or false has a determinate answer as soon as the element P and the sequence in question are themselves determined, (2) if the element P is the limit of the sequence $P_1, P_2, P_3, \ldots$ and $n_1, n_2, n_3, \ldots$ is an infinite sequence of positive integers such that $n_1 < n_2 < n_3 \ldots$, then P is also the limit of the sequence $P_{n_1}, P_{n_2}, P_{n_3}, \ldots$, (3) if P is an element of L, P is the limit of the sequence $P, P, P, \ldots$, whose elements all coincide with the element P. An element P is said to be a *limiting element* of a sub-class M of the class L if P is the limit of some infinite sequence of distinct elements belonging to M. The set M is said to be *compact* if every infinite set of distinct elements belonging to M has at least one limiting element. The totality of all the limiting elements of a given set M is called the derived set of M. A class S is a class L in which the derived set of every set is closed. An element P belonging to L is said to be *interior to* the sub-class M of the class L if M contains P and at least one element of every sequence of distinct elements that converges to P. A family G of sub-classes of a class L is said to *cover* a subclass M of the class L if every element of M is interior to some member of the family G. A set of elements M is said to possess the Heine-Borel property if for every *countably* infinite family G of sub-classes of L that covers M there is a finite sub-family of G that also covers M. A sub-set M of L is said to possess the Heine-Borel-Lebesgue property if for *every* family G of sub-classes of L that covers M there is a finite sub-family of G that covers M.

In a recent paper[3] Fréchet has shown that in order that, in a given class S, a point-set[4] M should have the Heine-Borel property it is necessary and sufficient that the set M should be closed and compact. He also shows that the same conditions are necessary and sufficient in order that a point set M in a class V[5] should have the Heine-Borel-Lebesgue property. The Heine-Borel Theorem or the Heine-Borel-Lebesgue Theorem is said to hold true in a given space L if in that particular space every closed and compact point-set has the Heine-Borel property or the Heine-Borel-Lebesgue property respectively. Fréchet points out that in order that the Heine-Borel-Lebesgue Theorem should hold true in a given class L it is necessary, but not sufficient, that the class L should be a class S; and sufficient, but not necessary, that it should be a class V. He raises the question as to what property it is *necessary and sufficient* that a class L should possess in order that the Heine-Borel-Lebesgue Theorem should hold true in that class. In the present paper I will exhibit one such property.

I will call a family G of point-sets a *monotonic family* if, for every two point-sets of the family G, one of them is a subset of the other one. A sub-class M of a class L will be said to have the property K in case it is true that for every monotonic family G of closed sub-classes of M there is at least one point which is common to all the members of the family G. A class S in which every compact subset has the property K will be called a class S^*.

Lemma 1. *If in a class L every set of points which contains the point P in its interior contains at least one point of the set M distinct from P, then the point P is a limiting element of the set M.*

Proof. If P were not a limiting element of M then it would be interior to $L - M + P$. But this set of elements contains no point of M distinct from P.

Theorem 1. *If, in a class S^*, M is a closed and compact set of points and β is a well-ordered sequence of point-sets such that M is covered by the family composed of all the members of β, then there exists a member g of the sequence β such that M is covered by the family composed of g together with all those members of β that precede g.*

Proof. Suppose there exists no such member g. Then for each member x of the sequence β let M_x denote the set of all points P belonging to M such that P is not in the interior of x or of any member of β that precedes x. For every x, M_x contains at least one point. For every x, M_x is closed. For suppose P is a limiting element of M_x. If P were not in M_x then it would necessarily be in the interior either of the point-set x or of some point-set of the sequence β that precedes x and therefore, by a lemma of Hedrick's,[6] this particular point-set would contain, in its interior, at least one point of M_x, which is contrary to the definition of M_x. It follows that the family composed of all M_x's for all members x of the sequence β is a monotonic family of closed point-sets. Hence there exists a point P_0 which is in every M_x. But P_0 is in the interior of some point-set x_0 of the sequence β. Hence P_0 is not in M_{x_0}. Thus the supposition that Theorem 1 is not true leads to a contradiction.

Theorem 2. *In order that the Heine-Borel-Lebesgue Theorem should hold true in a given class S it is necessary and sufficient that that class S should be also a class S^*.*

Proof. Suppose that in a given class S^* the closed and compact point-set M is covered by the infinite family G of point-sets. The members of the family G can[7] be arranged in a well-ordered sequence β. By Theorem 1, there exists at least one member g of β such that M is covered by the family composed of g and all those members of β that precede g, together with not more than a finite number of those elements of β that follow g. Let g_1 denote the first[8] such g and let $g_2, g_3, g_4, \ldots g_n$ denote a finite set of members following g_1 such that M is covered by $g_1, g_2, g_3, \ldots g_n$ together with all those members of β that precede g_1. There clearly cannot exist a member of β immediately preceding g_1. I will show that g_1 is the first member of β. Suppose it is not. Then if g_1 and all the succeeding members of the sequence β are removed, there remains a well-ordered sequence β_1 whose members are the remaining members of β arranged in exactly the same order as in the original sequence β. Let us now construct a third well-ordered sequence β_2 having for its first n members the point-sets $g_1, g_2, g_3, \ldots g_n$ arranged in the order indicated and having as its remaining members the members of β_1 arranged in the same order as in β_1. The point-set β_2 has no

last member. The point-set M is covered by the family of point-sets composed of all the members of the sequence β_2 but there does not exist any member g of β_2 such that M is covered by the family composed of g together with all those members of β_2 that precede g. But this is contrary to Theorem 1. Thus the supposition that g_1 is not the first member of β has led to a contradiction. It follows that g_1 is the first member of β and that M is covered by the finite set of point-sets $g_1, g_2, g_3, \ldots g_n$. Thus the Heine-Borel-Lebesgue Theorem holds true in every class S^*.

Suppose now that the Heine-Borel-Lebesgue Theorem holds true in a given class S and that G is a monotonic family of closed, compact subsets of S. Let $\bar{g}$ denote any point-set of the family G. I will show that the members of G have at least one point in common. Suppose that this is not the case. Then, if P is a point of $\bar{g}$, there exists a closed point-set g_P of the family G that does not contain P. Hence, by Lemma 1, the point P is in the interior of some point-set R_P which contains no point of g_P. Let H denote the set of all R_P's for all points P of $\bar{g}$. By hypothesis, $\bar{g}$ is covered by a finite subfamily $R_{P_1}, R_{P_2}, R_{P_3}, \ldots R_{P_n}$ of the family H. But there exists an i $(1 \leqq i \leqq n)$ such that g_{P_i} is a subset of each of the point-sets $g_{P_1}, g_{P_2}, g_{P_3}, \ldots g_{P_n}$. It follows that no point of g_{P_i} is in any one of the sets $R_{P_1}, R_{P_2}, R_{P_3}, \ldots R_{P_n}$. Thus the supposition that there is no point common to all the members of G has led to a contradiction.

§2. I now raise the question whether it is not desirable to substitute, for Fréchet's definition of the word compact, a definition which is, for some spaces, (but not for spaces V) more restrictive than that of Fréchet. I will say that a monotonic family of point-sets is *proper* if there is no point that is common to all of its members. I will say that a set of points M is *compact in the new sense*[9] if for every proper monotonic family F of subsets of M there exists at least one point which is a limit point of every point-set of F.

Suppose that in a space L the infinite point-set N is compact in the new sense. The set N contains at least one countably infinite sequence of distinct points $P_1, P_2, P_3, \ldots$ For each n let t_n denote the point-set $P_n, P_{n+1}, P_{n+2}, \ldots$ The family of point-sets $t_1, t_2, t_3, \ldots$ is a proper monotonic family. Hence there exists a point P which is a limiting element of every one of these point-sets. It follows that if in a space L a point-set is compact in the new sense then it is also compact in the sense of Fréchet. That the converse is not true for every space S may be seen with the help of the following example.

Example. Let α be a well-ordered set of elements such that [10] (1) if K is any countable subset of the elements of α then there exists an element of α that follows all the elements of K, (2) if P is a given element of α the set of all those elements of α that precede P is countable. If a and b are two non-consecutive elements of α such that a precedes b then the set of all those elements of α which follow a and precede b will be called a segment. An element P of α will be said to be the *limit* of a countable sequence $P_1, P_2, P_3 \ldots$ of ele-

ments of α if, and only if, for every segment s that contans P there exists a positive integer δ_{PS} such that, for every n greater than δ_{PS}, P_n is in s. With respect to this conception of limit of a sequence, the elements of α evidently constitute a class S. The set M composed of all the elements of α is compact in the sense of Fréchet. It is not, however, compact in the new sense. For if for every element x of α, t_x denotes the set of all those elements of α which follow x then there exists no-element which is a limiting element of every member of the proper monotonic family composed of all t_x's for all elements x of α. The set M, though closed and compact in the sense of Fréchet, does not possess the Heine-Borel-Lebesgue property.

It is easy to see that, in every class V of Fréchet, a point-set which is compact in the Fréchet sense is also compact in the new sense.[11]

By a proof in large part identical with the above proofs of Theorems 1 and 2, the truth of the following theorem may be established.

Theorem 3. *In a class S, in order that a point-set M should possess the Heine-Borel-Lebesgue property it is necessary and sufficient that M should be compact in the new sense and closed.*

[1] Fréchet, M., *Palermo, Rend. Circ. Mat.*, 22, 1906, (1–72).

[2] By a countable sequence is meant a sequence of the same order type as the sequence of positive integers arranged in the normal order.

[3] Fréchet, M., *Bul. sci. math., Paris*, 45, 1917, (1–8). See also Chittenden, E. W., *Bull. Amer. Math. Soc., New York*, 25, 1918, (60–65).

[4] In the present paper the elements of a class L will be called points.

[5] A Class V is a class L in which there exists a distance function. Cf. either of the above mentioned papers of Fréchet.

[6] Hedrick, E. R., *Trans. Amer. Math. Soc., New York*, 12, 1911, (285–294). Cf. also Fréchet, loc. cit.

[7] For a proof that the elements of any set (whether countable or uncountable) can be arranged in a well-ordered sequence, see Zermelo, E., *Math. Ann., Leipzig*, 59, 1904, (514) and 65, 1908, (107–128). Zermelo assumes the truth of the well-known Zermelo Postulate. With regard to this postulate, see a recent paper by Ph. E. B. Jourdain, *Paris, C. R. Acad. Sci.*, 166, 1918, (520 and 984).

[8] Every subset of a well-ordered sequence contains a first member, that is to say, a member that precedes every other member of that subset.

[9] It is clear that in every space S a point-set is compact in the new sense if and only if, it possesses the property K defined in §1 of the present paper.

[10] For a proof of the existence of a well-ordered sequence satisfying these two conditions cf. Hobson, E. W., *The Theory of Functions of a Real Variable*, Univ. Press, Cambridge, 1913, (177–181).

[11] Sometime after the manuscript of the present paper had left my hands I found that, in 1912, S. Janiszewski introduced an extended conception *of limit* and defined a point-set as "*parfaitement compact*, si de toute de ses éléments on peut extraire une suite du même type d'ordre et possédant un élémente limite." cf. J. éc. polytech., Paris, 16, 1912 (155). Compare also the example in §2 of the present paper with an example of Janiszewski's on page 167, loc. cit. It seems likely that in a class S a set is compact in my sense if and only if it is parfaitement compact in the sense of Janiszewski. I do not find however that Janiszewski has made any study of the Heine-Borel-Lebesgue Theorem in connection with his conception of compactness.

AN ELECTRODYNAMOMETER USING THE VIBRATION TELESCOPE

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Communicated, April 21, 1919

1. Introductory.—The employment of a telescope with a vibrating objective did good service as an aid to the interferometry of vibrating systems. It seemed worth while therefore to see what could be got out of it, when used in connection with a telephone only, as a dynamometer. The experiments are of interest both because of the vibratory phenomena observable and in view of the peculiar method of optic observation developed. Its possible use for finding the magnetic field within a helix of unknown constants deserves mention.

2. Apparatus.—A front view (elevation) of the design is given in figure 1 and an enlarged detail (side view) in figure 2. The apparatus consists of a rigid rectangular frame-work of ½ inch gas pipe, A, B, C, D, EE', F being the foot attached to a tripod. There may be a steadying foot at C'. A and D are attached to EE' by the stout clamps c, c'', so that EE' lying behind the plane of $ABCD$, admits of the attachment of a suitable clamp c', by which the telephone ih may be held in the same plane. B and C may be forced apart slightly by the screw n controlled by the broad thumb nut m, the conical end of n rotating in a socket of the cap p.

The vibrating system consists of the bifilar wires of phosphor bronze dd' and the frame of the lens f which is the movable objective of the telescope T, the latter part containing the ocular and a plate micrometer (cm. divided in 100 parts). T may be at a considerable distance (50 cm. or more) from f, and supported by a convenient standard. The frame of the lens (which must hold it securely, cement being used if necessary) is of light sheet metal, the parts gg' being of sheet iron (about 0.05 cm. thick) so as to be attracted by the magnet, i, of the telephone. The stiff cross wires, r,s, of the frame are either soldered to the bifilar system dd' or otherwise attached to it (soft sealing wax does very well for temporary experimental purposes).

The attachment and tension-control of the bifilar suspension is finally to be described, as its period must be synchronized with the alternating current. Results are obtainable only when the two periods are strictly in unison. In figure 1 the wires dd' are looped around a groove in the pipe D below, and the upper ends of dd' after passing a similar groove in A are bent around the post a, a', and wound respectively around the snugly fitting screws, b, b', the ends being secured against sliding by a fine hack saw cut in the screws. To stretch a wire it is passed from the notch in b once or twice around it, thence around a, downward by the groove to D and then up in the corresponding way to b'.

The lens carriage gg' is then attached with cement (after the wires are evenly stretched) with the crosswires r, s on the opposite side of dd' to the pull of the magnet i. The magnet, in addition to the cement, thus guards against slipping. On turning the screws b and b' any degree of tension may be imparted to the wires bb', roughly. This simple device worked surprisingly well, and wires of different kinds may be easily inserted or replaced, the lens system being subsequently attached with cement; but it is better to loop the lower part of dd' around a special roller G, as indicated in figure 2, and used in my later tests, with the object of more easily reaching an equality of tension in the wires d and d'. The tensions are then roughly changed by the screw and nut u.

The approximate tension having thus been obtained by the screws u, b, b', the finer variations are imparted by the screw mn which flexes the elastic rectangle $ABCDE$ and thus gives to the bifilars d, d' exactly the tension required. It is at the thumb nut m that all adjustment is made during observation.

In my apparatus the rectangle was about 50 cm. long and 12 cm. wide. The wires d, d' about 1.5 cm. apart and each about 0.025 mm. in diameter. Wires as thick as this require sharp adjustment as to tension, but the method given proved quite satisfactory particularly as it is little disturbed by manipulation. The tension is sufficient to admit of an air gap of less than a millimeter between the plate g' and the magnet i of the telephone. Later the telephone was also put on a spring-micrometer screw for fine adjustment.

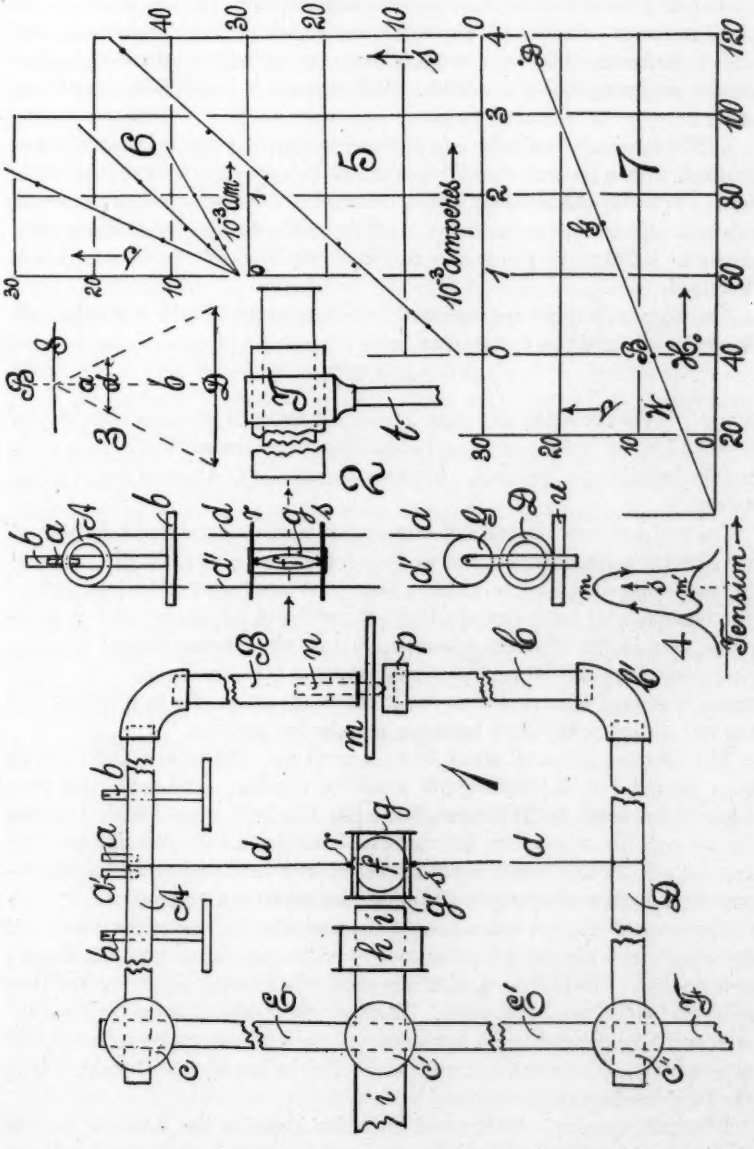
In the case of parallel rays, the displacement of the image in the ocular would be no larger than the displacement of the objective, f, figure 1. To obtain increased displacement, the method of figure 3 is available where S is the fine slit in front of a Welsbach burner at B. At d is the principal plane of the vibrating objective and at D the micrometer plate in the ocular. Again, if the length d represents the double amplitude of the objective and the sides of the triangle be drawn from S, the intercept D will represent the displacement in the ocular. If the distance Sd be a and dD, b, we may write

$$1/a + 1/b = 1/f \tag{1}$$

where f is the principal focal distance of the objective. Hence

$$D/d = b/f \tag{2}$$

Theoretically, therefore, any degree of magnification is possible by increasing b (the distance of T and f, fig. 2) and decreasing f. In the former case, some means of controlling the thumb nut m, figure 1, from a distance would have to be provided. In the latter the lens f would have to be achromatic. In the present experiments I first used an ordinary spectacle lens at f with a slotted screen between it and the slit to diminish chromatic aberration; but there is no objection to the use of an achromatic lens at f, as was done later, particularly since a breadth of a few millimeters of lens will suffice, for there is an abundance of light.


Tension
10^{-3} amperes
10^{-3} cm
1
2
3
4
5
6
7

To measure the width of the image band of light produced by the vibration, the ocular T should be on an axle t with slight friction. In my earlier improvised apparatus, single scale-parts (0.1 mm.) only could be guaranteed; but with a perfected optical system there is no reason why tenths of scale-parts should not be equally trustworthy. This makes a scale of 1000 parts in the ocular.

3. Observations.—As a source or alternating current I selected a small induction coil with a mercury break (Kohlrausch's design) to facilitate the initial tests. This was put in series with a rheostat (to 30,000 ohms), a Siemens's precision dynamometer reading to within milliamperes, an ordinary telephone to indicate the continuous action of the coil and the vibrator above described.

The Siemens's dynamometer was first standardized with a Clarke cell. Accepting the effective current i as being

$$i = C\sqrt{\varphi}$$

where φ is the deflection on a scale at about 1 meter of distance, the constant $C = 1.12 \times 10^{-3}$ relative to amperes was found and the mean resistance of the coils included about 250 ohms. Virtual currents of 10^{-4} amperes would escape detection.

The coil was now started and measurements made simultaneously both at the Siemens's dynamometer and at the vibrating telephone (slit distance a = 35 cm., ocular distance b = 75 cm.), with good results, the virtual currents of the dynamometer being compared with the width of the image band (in scale-parts) seen in the vibration telescope. To obtain different virtual currents, resistances from 10,000 to 2000 ohms were put into the circuit. The dynamometer showed deflections from 2 to 20 cm. on the scale. For larger deflections the coil current would have been too irregular for use.

The vibrator indicated about 10 scale-parts per milliampere, and readings even beyond 5 or 10 milliamperes would be possible. The deflections were fairly proportional to the current, indicating that with a good optical system virtual currents as small as 10^{-5} ampere should have been perceptible. The apparatus is thus very well worth further development and would be particularly useful where alternators with a definite period are in question.

The method of observation consists in gradually increasing the tension of the wire from a slightly low value, to beyond the maximum tension. In this case the band widens from a relatively small width to the maximum and then abruptly falls off to a small value. To repeat the observation the tension must often again be reduced to the low value and the whole operation repeated; but after some practice maximum may be reached in the reverse direction unless the band has become too narrow.

When the current is broken and thereafter closed, a low width of band is obtained which will not widen unless the operation described is carried out from low tension. In other words there are often two cases of equilibrium for each

current, corresponding to very different band widths. This is a curious result, for it means, virtually, that the magnetic forces and the stresses are in the relation of a doubly inflected curve to each other, so that there are three intersections, two for stable vibratory equilibrium; or else the two harmonic systems, the electrical and the mechanical, may vibrate in the same or in opposed phases. The cycle in figure 4 indicates the general relation of the band width s to the tensions, there being two maxima at m and m'.

Similarly each current requires its own particular maximum tension, which increases with the current. The difference is not large, but effective and for this reason the fine tension adjustment is essential.

4. Further observations.—The apparatus was then improved in a variety of ways, chiefly by the insertion of a small vibration objective, about 1 cm. in diameter, achromatic and with a focal distance of but 5.8 cm. In this case the distance a and b, figure 3, could be decreased to 7 cm. and 35 cm. and the observer was thus conveniently near the adjusting screw. The slit image was white and about a scale-part in width. There would have been no difficulty in using much greater magnification.

An example of the results (band-width s in scale parts) is given in figure 5. The constant of the dynamometer was now $C = 10^{-3} \times 0.87$ relative to amperes and the total resistance in circuit 710 ohms. The frequency, as before, was estimated at about 15 per second.

The results were a considerable improvement on the preceding and the discrepancies as a rule lie within 5×10^{-5} ampere. They are much more liable to be in the dynamometer than in the vibrator, as the former was not well adapted for these small currents. The deflections begin with 3 scale-parts (initial slit breadth) and not at zero; but as this appears merely as an initial constant, it is not of consequence.

If we compute the coefficient of induction as

$$L^2\omega^2 = \Delta(i^2(R + r)^2)/\Delta i^2$$

(Δ being a differential symbol) from the first and fifth, second and sixth, etc., observations, data for $L\omega$ and L follow.

5. Effect of frequency.—A special mercury interruptor was now made having as its distinctive feature contrivances by which the mercury surface was washed and thus could always be kept clean and bright. It was furthermore adapted to give different frequencies. The apparatus functioned admirably for days, frequent washing presupposed. Different frequencies, obtained by a sliding weight, were estimated from the moments of inertia as $n = 10$, 15, and 20. The latter could just be counted in groups of 4 vibrations with a stop-watch. Higher frequencies were obtainable by using stiffer springs.

An example of the results obtained with this apparatus is given in figure 6, for the phosphor bronze bifilar differently stretched. All gave evidence of the peculiar fact that the sensitiveness increases in marked degree with the frequency.

It is difficult to account for this effect of frequency, so peculiarly marked in the instance given, where the observations were good. If different harmonics are in action, the overtones would have to respond in case of the wires under less tension, and this seems unlikely. The behavior of very tense steel wires, moveover, was the reverse of this. It is probable that the phenomena of §6 contain the clue.

6. Adjustable telephone.—The question as to the most advantageous position of the telephone is thus variously important. Consequently the telephone *ih*, figure 1, was mounted on a stout flat steel spring controlled by a micrometer screw. By actuating this, the face near g' could be approached as near g' as permissible, or withdrawn to a remoter position, with precision. The experiments made at length showed that within a wide range of tensions and of telephone positions, a particular degree of approach of the telephone corresponded to each particular stress of wire. Unless these paired positions are selected, the bifilar system does not respond. Tense wires require a nearer telephone; less tense wires a more remote telephone (even 0.5 cm.), within wide limits. Therefore the condition of resonance may be reached either by adjusting the telephone on the micrometer screw for a given tension of wire; or by changing the tension of the wires for a fixed telephone. Between the admissible limits of tension, the sensitiveness passes through a flat maximum.

With a distance of not more than 50 cm. between slit and ocular, a judicious disposition of parts eventually gave me 40 ocular scale parts per virtual milliampere, viz.,

n	$i \times 10^3$	$s \times 10^3$
	amperes	*cm.*
15	0.67	27
20	0.67	27

so that here the above effect of frequency, n, is no longer apparent.

7. Coil tester.—An interesting application of the above apparatus, where a definite frequency is usually assigned, is its possible use for measuring the magnetic fields of different coils. For this purpose I wound a long thin primary of an induction coil, which when inserted into the coil to be tested, should, from the measurement of the current induced in the secondary in question and in the absence of other mutual inductions, give the constants of the secondary. As many of the coils to be tested were of small internal diameter, the primary was wound on a long iron tube, fine wire being necessary. The data are for the

Iron tube	*Helix*
Diameter outside.....0.635 cm.	Diameter outside.....0.7 cm.
Length .....55 cm.	Wire.....0.034 cm. in diameter
Walls.....0.08 cm. thick	n_1/l_1 .....21 turns per linear cm.

If L is the coefficient of induction per turn of primary, the total induction is

$$B = LN_1 i \tag{1}$$

Hence the electromotive force induced in the secondary becomes

$$e = LN_1N_2(di_1/dt) \tag{2}$$

If the field of the secondary per unit current is put

$$H_2 = 4\pi N_2/l_2 \tag{3}$$

for N_2/l_2 turns per linear centimeter, and r the resistance of this coil and its circuit, we may compare any two coils by the equation

$$\frac{e_2}{e_2'} = \frac{N_2}{N_2'} = \frac{H_2 l_2}{H_2' l_2'} = \frac{i_2 r_2}{i_2' r_2'} \tag{4}$$

e_2 and e'_2 being the electromotive forces, and i_2 and i'_2 the currents induced in the two secondaries in question. Thus

$$\frac{H_2}{H_2'} = \frac{i_2 r_2 / l_2}{i_2' r_2' / l_2'} = \frac{s_2/l_2}{s_2'/l_2'} \tag{5}$$

it being assumed that the resistance r in the secondary circuit is made so large that the inductive resistance vanishes.

The coil tester was now thrust through a variety of helices, differing in shape and construction.

Figure 7 gives the results read off for these coils at the vibrator (s), when a high resistance was added to the circuit and Ri constant. The relation of H_0 and s as seen in figure 7, is linear for coils W, B, G, which were about of the same length and wound on non-conductors or split brass. The result for coil D, which was about 1.8 times longer is low, probably owing to mutual induction, as this coil was wound on a thick brass tube without fissure. Similarly it made very little difference whether the two helices wound side by side throughout G, were used in parallel or in series. Finally a number of single layer coils of lengths 10, 20, 30, 40 cm. were wound on stout glass tubing and compared with B. Provided care was taken to prevent the induced currents from heating the telephone, the band width s increased throughout proportionally to the number of turns of wire in the secondary and under good conditions at a rate of about $s = 0.06$ scale parts per turn of wire.

THE TEMPERATURE OF THE HUMAN SKIN

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Read before the Academy, April 28, 1919

The clinical importance of records of body-temperature, as taken usually in the mouth and occasionally in the axilla or rectum, has quite obscured the physiological significance of the skin temperature. Extensive researches have shown that the temperature of the human body, deep in the body trunk or in any of the natural cavities, remains reasonably constant, although there is a diurnal ryhthm, with a minimum value at about 4 a.m. and a maximum at about 5 p.m. Simultaneous observations of temperature at different parts of the body show that there is almost always a parallelism in curves for temperature. Thus, the temperatures in the rectum or vagina, the well-closed groin or axilla, and the mouth, show similarly shaped curves, although at markedly different levels.

The technical difficulties in recording skin temperature have undoubtedly retarded extensive study in this phase of body-temperature research. The temperature of the human skin is the resultant of several factors, as heat is supplied from the subcutaneous tissues and lost from the surface of the body by radiation, conduction, and the vaporization of water. An attempt to secure accurate records of skin temperature by the application of an ordinary mercury thermometer is obviously useless, for but a small proportion of the bulb of such a thermometer comes in actual contact with the skin. Even thermometers constructed with a special bulb providing a large surface to apply to the skin have a like surface exposed to the environmental temperature. If, as is occasionally done, this outer surface is covered with non-conductive material, there is almost immediately a disturbance in the temperature of the skin due to the fact that there is a retardation of the normal loss of heat, with a consequent accumulation of heat from the subcutaneous tissue. The true temperature of the skin should therefore be recorded by an apparatus which is nearly instantaneous in action and sufficiently protected from the environment to insure a true record of the surface temperature, and not a resultant of skin and environmental temperature.

A method recently used at the Nutrition Laboratory consists of two copper-constantan junctions, one located in a constant-temperature bath—a Dewar flask—and the other applied to the skin. The resulting current which can be measured on any one of several types of galvanometers is directly proportional to the temperature difference between the two junctions. All thermometric lag is thereby eliminated and it is only necessary that the junction which is applied to the body should be properly protected from the environmental temperature. It was found that when this junction was backed with

a light tuft of cotton batting and installed rigidly in a piece of hard rubber, it was possible to apply it to the body and have it assume the surface temperature inside of a few seconds. Thus the period of application was so short that the protecting material did not sensibly affect radiation, conduction, or the vaporization of water.

In our ordinary skin temperature measurements, the junction in the constant temperature bath is kept at 31° or 32°C. When the other junction is exposed to usual room temperature, there is a large deflection of the galvanometer. On applying the junction to the surface of the skin, the amplitude of deflection decreases rapidly and with negative acceleration. The deflection level is reached in about six seconds after application of the junction. Thereafter the rate of movement of the galvanometer is extremely slow, being but a millimeter or two for each successive minute. The explanation of this is that the junction almost instantly assumes the true skin temperature. Thereafter the skin temperature gradually rises, as the protection of the junction with cotton batting and hard rubber stops the loss of heat by radiation, conduction, and vaporization of water, and there is then a 'building up' of the temperature due to the subcutaneous source of heat. As a matter of fact, observations taken for some time show that this rising temperature continues, the length of time depending in large part upon the amount of hard rubber and cotton batting used. For all of our work we have assumed that under these conditions the true skin temperature is obtained at the end of about six seconds after application of the junction.

In connection with an extended series of metabolism experiments on the influence of temperature environment upon the metabolism, it was found that the subject used (a professional artist's model) could withstand exposure to temperatures as low as 14°C. for several hours without shivering. These conditions presented unusual opportunities for studies of skin temperature which would give evidence first, as to the rapidity of the change on exposure of the body to the environmental temperature, and finally as to the absolute level which the skin temperature reaches after prolonged exposure to an environment varying from 14° to approximately 30°C.

When the subject arrived at the Laboratory, the clothing was loosened and temperature measurements were made at numerous points on the skin under the clothing. Prior to these records, the subject clothed and wearing a heavy coat had been exposed to an external temperature of about 17°C. The results of one series of measurements are recorded in table 1. The extreme range in the skin temperature on this particular day was 6.6°C. Special attention is called to the relatively high value of 31.6°C. on the exposed forehead, the record of approximately 34°C. at the waist, a part well protected by clothing, and values of 30°C. or below on the buttocks, shin, and calf. This series is typical of the skin temperature of a normally-clothed woman.

The method of using isolated points, such as those given in table 1, can be much improved upon by taking advantage of the fact that the thermal junction, when not excessively protected by the cotton batting and hard rubber, almost instantaneously assumes the skin temperature. This is particularly the case if the junction is moved from one part of the skin to another after the hard rubber and cotton have assumed approximately the temperature of the skin. When recording the values of isolated points, we have employed a good grade, direct reading galvanometer, thus making the apparatus compact and portable. The response of the thermal junction is so rapid, however, that when connected with a sensitive aperiodic instrument, i.e., a string galvanometer, it is perfectly possible to move the junction at a moderately rapid rate over the surface of the body and record photographically a continuous and true temperature curve for an infinite number of points on the skin.

TABLE 1

RECORDS OF SKIN TEMPERATURE UNDER THE CLOTHING

MEDIAN LINE		RIGHT SIDE		LEFT SIDE	
Location	°C.	Location	°C.	Location	°C.
Front		*Front*		*Front*	
Forehead	31.6	Neck	32.5	Neck	32.1
Second rib	30.3	Nipple line		Nipple line	
Fourth rib	32.1	Upper chest	33.6	Upper chest	33.7
Lower edge breast bone	33.2	Under breast	33.8	Under breast	33.7
7.0 cm. above umbilicus	34.4	Waist	34.4	Waist	34.7
3.0 cm. below umbilicus	33.4	Groin	34.0	Groin	32.8
11.5 cm. below umbilicus	31.8	Thigh	30.7	Thigh	31.1
		Shin	29.8	Shin	30.9
		Back		*Back*	
		Shoulder blade (above)	33.6	Shoulder blade (above)	33.7
		Shoulder blade (below)	32.5	Waist	33.7
		Waist	33.7	Buttock	29.9
		Buttock	30.5	Thigh	31.4
		Thigh	31.0	Calf	28.2
		Calf	28.1	Hand	32.3
		Hand	32.5		

To illustrate our many results obtained by this latter method, two typical curves are given in figure 1. These were obtained after the nude subject had been exposed to an environmental temperature of 14.6°C. for two and one-half hours, the greater part of the time in the standing position. The upper curve follows the left mammillary line, while the lower curve was taken in a corresponding position on the back. An extraordinary difference in skin temperature at the different points may be noted, the extreme range on the front and back showing differences of 10°C. or more. The rectal temperature taken simultaneously was 36.7°C.

The pronounced influence of the environmental temperature upon the curve for the skin temperature was especially studied by making temperature photographs over exactly the same line at intervals from the moment of disrobing

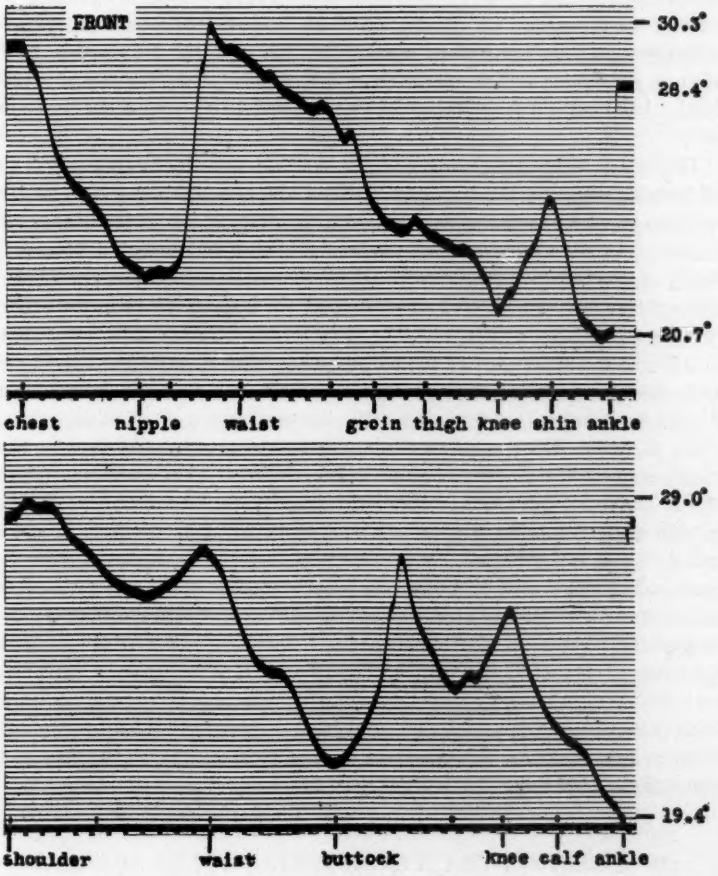

FIG. 1

Photographic records from a string galvanometer for skin temperatures at different parts of the nude body following a 2½ hour exposure to an environmental temperature of 14.6°C. The thermal junction was moved at constant tempo from shoulder to ankle. Time registered at bottom of each curve is in 2-second intervals.

to the end of the period of prolonged exposure to cold. As would be expected, these show a progressive increase in the amplitude of the curve as the time of exposure is prolonged; marked changes occurring even inside of the first fif-

teen minutes. Similar studies made at the end of the exposure to environmental temperatures varying from 14° to 30°C. show a distinct tendency towards a flattening out of the curves at the higher temperatures. Thus when the thermal junction was passed down the front and back of the body over exactly the same lines and at the end of two and one-half hours' exposure, the difference between the highest and lowest points in the curves was 10.6°C. with an environmental temperature of 14.6°C.; 9.8°C. with a temperature of 19°C.; 5.4°C. with a temperature of 25.8°C.; and 4.2°C. with a temperature of 30°C.

This study of the temperature of the skin has two important bearings upon all investigations on the heat production of the human body. First, in all researches on direct calorimetry it has been necessary to corrct the heat actually measured by the calorimeter for the heat gained or lost from the body as the result of changes in temperature. Heretofore it has been assumed that as temperature curves measured either deep in the body trunk or in the artificial cavities are similar, a change of 0.1° in rectal temperature indicates a change of 0.1° in the temperature of the entire body. Our observations, particularly with cold temperature environments, show skin temperatures falling several degrees even when interior body-trunk temperatures may be simultaneously rising slightly. The correction of direct heat measurements by records of the rectal temperature is thus open to grave criticism. Unfortunately no substitute correction can as yet be offered. Secondly, these pronounced differences in skin temperature have great significance in any consideration of the so-called 'law of surface area.' It is still maintained by many physiologists that, practically independent of species, the heat production of the quiet organism is determined by its surface area. Our observations show clearly that, contrary to popular impression, the temperature of the skin, presumably one of the most important factors affecting heat loss, is very far from uniform for we have seen that even with well-clothed individuals differences in the temperature of various localities of 4° to 5°C. are of regular occurrence. These observations bring to light and establish one more serious objection to the legality of the conceptions underlying the 'body-surface law.'

ON A CERTAIN CLASS OF RATIONAL RULED SURFACES

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Communicated by R. S. Woodward, May 19, 1919

As is well known, ruled surfaces, or scrolls as Cayley calls them, may be generated or defined in a number of ways. There exists, for example, a one-to-one correspondence between ruled surfaces and a certain class of partial differential equations, so that the theories of the two classes are abstractly identical.

A much favored method, especially in descriptive geometry, consists in considering ruled surfaces as continuous sets of straight lines, or generatrices, which intersect three fixed curves, the directrices, simultaneously. If these are algebraic curves of orders l, m, n, with no common points, the ruled surface which they determine is, in general, of order $2\,l.\,m.\,n$.

Frequently, ruled surfaces are also defined as systems of elements, either common to two rectilinear congruences, or to three rectilinear complexes.

Of great importance for the following investigation is the definition of ruled surfaces as systems of lines which join corresponding points of an (α, β) — correspondence between the points of two algebraic curves C_m and C_n of orders m and n. If these curves are plane, and if to a point of C_m correspond α points on C_n, and to a point of C_n β points of C_m, then the order of the surface is in general $\alpha\, m + \beta\, n$.

Finally there is the cinematic method in which ruled surfaces are generated by the continuous movement of the generatrix according to some definite cinematical law. In this connection the description of the hyperboloid of revolution of one sheet is well known.

The literature seems to contain but little about this method of generating ruled surfaces. A number of treatises on differential geometry contain chapters on cinematically generated surfaces.

The class of surfaces here considered is obtained as follows: Given a directrix circle C_2 and a directrix line C_1, which passes through the center of C_2 at right angles to the plane of C_2. The generatrix g moves in such a manner that a fixed point M of g moves uniformly along C_2, while g in every position passes through C_1. The plane e through C_1 in which g lies evidently rotates about C_1 with the same velocity $k\theta$ as M. In this plane e, g rotates about M with a uniform velocity $k\mu\theta$. When $\mu = p/q$ is a rational fraction, the surface generated is also rational and belongs to the class of ruled surfaces generated by means of an (α, β) correspondence between C_1 and C_2.

When C_1 coincides with the z-axis, so that C_2 lies in the xy-plane, and we denote by ρ the distance of the projection P' of a point P on the generatrix g from the origin and by θ the angle $P'OX$, the equations of the surface expressed by the parameters ρ and θ are

$$x = \rho \cos\theta,\ y = \rho \sin\theta,\ z = (\rho - a) \cot\frac{p}{q}\theta.$$

It is shown that these may be expressed rationally by ρ and another parameter t. Also the implicite cartesian equation of the surface is obtained, as well as are the parametric equations of the double curve of the surface. The following theorems are of interest:

Theorem 1. The surface of the class is rational and of order $2\,(p + q)$ or $p + q$, according as q is odd or even.

Theorem 2. When q is odd the generatrices of the surface cut C_1 and C_2 in two

point sets which are in a $(q, 2p)$ *— correspondence.* C_1 *and* C_2 *are 2 p-fold and q-fold curves of the surface. The surface has moreover p real and* $2pq - 2p - q + 1$ *imaginary double generatrices.*

When $q = 2s$ *is even the generatrices cut* C_1 *and* C_2 *in two point sets which are in an also rational* (s, p) *— correspondence.* C_1 *and* C_2 *are respectively p- and s-fold curves of the surface. The surface has no real, but* $ps - p - s + 1$ *imaginary double generatrices.*

In the whole discussion the assumption is made, of course, that p and q are relatively prime.

Theorem 3. When q is odd the order of each of the $(q - 1)/2$ *double curves is 4 p or 2 q according as* $q \lesseqgtr 2p$. *They are rational and each lies on a surface of revolution of order 4 generated by the rotation of an equilateral hyperbola about the z-axis.*

Theorem 4. When $q = 2s$ *is even and s odd, there are* $(s - 1)/2$ *double curves of order 2p or q according as* $p \gtrless s$, *and one double curve of order p or s, according as* $p \gtrless s$. *When* $s = 2\sigma$ *is even, there are* σ *double curves of order 2 p or q, according as* $p \gtrless s$.

The intersections of the double curves with a plane through the z-axis may be arranged according to certain cyclic groups whose orders may be easily determined. One interesting fact is that the surfaces of the class in certain species, are applicable among themselves. The following theorems appertain to this fact:

Theorem 5. Surfaces of the class are applicable to each other when their orders are $2(p + q)$ *and* $2(m p + n q)$, *and the ratio of the radii of their* C_2*'s is* m/n, *with q odd, p and q, m and n, and m and q as relative primes.*

Theorem 6. Surfaces of the class of odd order are applicable to each other when their orders are $p + q$ *and* $m p + n q$, *and the ratio of the radii of their* C_2*'s is* m/n. *Moreover q is even, and p and m are odd.*

As the surfaces of even and odd orders are respectively bifacial and unifacial, we have

Theorem 7. Bifacial and unifacial surfaces of the class are applicable to surfaces of the same type only.

The intersection of a torus with the surfaces of the class yields all so-called cycloharmonic curves. Also the 'bands of Moebius' may readily be cut out from the surfaces.

Among the class here considered are cubic, quartic, and quintic scrolls investigated by Cremona, Cayley, Schwarz and others. Models of these have been and are being constructed in the mathematical model shop of the University of Illinois.